Trees in focus
Practical Care and Management

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Shaded by Trees?

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Summary

The lengths of shadows cast by trees depends on factors which include the time of day and year and geographical location. The shape of tree will affect the duration of shading. Tables show the lengths of shadows for a range of parameters. These give quick reference having been chosen to give a representative range of locations in the British Isles.

Introduction

Shade can be highly desirable, providing a cool respite from hot summer sun. Equally, it can cause annoyance if, for example, the front rooms of a beautifully designed south-facing house become shaded by a row of dense conifers. Equal annoyance can occur when trees interfere with television reception - particularly reception from satellites (BBC and ITC 1998). Trees can be accommodated in landscape design by consideration of a few general and site-specific factors. This note explains these factors and provides easy reference tables for determining approximate shadow length at different locations in the British Isles.

Defining the problem

The length of a tree's shadow depends on the time of both year and day and the geographical location. Calculation of shadow length can be broken down into geometrical and time-dependent factors:

- The position of the sun in the sky (throughout day/year)
- The latitude of the location
- The immediate topography

The extent and duration of shading is also determined by the shape and height of the object, e.g. a tree. In order to simplify the problem, it is assumed that the tree is like a flag-pole and all that initially needs to be considered is its height. This assumption is reasonable because the width of the shadow and its duration at a point depends on the width of the tree's crown.

Where is the sun in the sky?

The sun rises in an easterly position and tracks through south to set in a westerly direction (Figure 1). Its path across the sky is an arc, determined by the Earth's tilt in relation to the Sun. Consequently, it reaches its maximum height at noon (local time) when, in the northern hemisphere, the shortest shadows of the day are cast due north of the tree. (Fig 3)

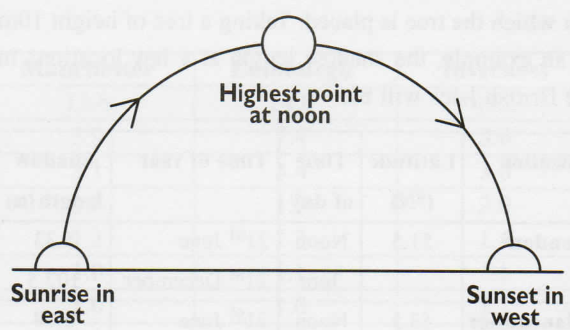

Figure 1.
Schematic diagram of the sun as it tracks across the sky during the day.

The length of a shadow is related to the tree's height and the position of the sun in the sky: (Fig 2)

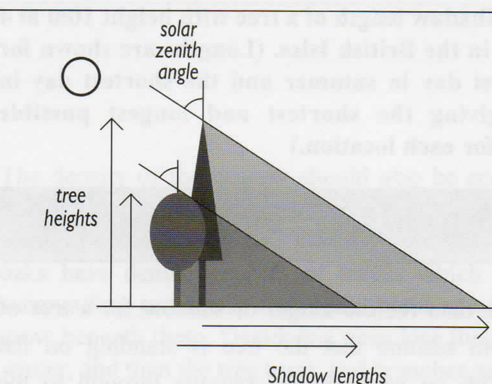

Figure 2.
The Shadow length is related to the height of the tree and the angle that the Sun makes with the sky zenith, i.e. the *solar zenith angle*

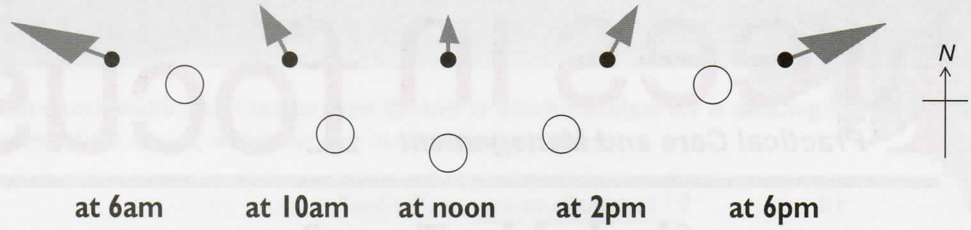

Figure 3.
The length and position of shadow throughout the day (as seen from above).

The *bearing* of the sun is the angle between north and its position in the sky - if it is due south then the bearing is 180°, due west is 270°, etc. The shadow is cast to the opposite side of the tree - to the north of the tree if the sun is in the south, for example. The longest shadows occur at sunrise and sunset, but at different bearings depending on the latitude and time of year.

The effect of latitude

Having considered the position of the sun in the sky, we next consider the latitude - distance north of the equator - at which the tree is placed. Taking a tree of height 10m as an example, the shadow length at a few locations in the British Isles will be:

Location	Latitude (°N)	Time of day	Time of year	Shadow length (m)
London	51.5	Noon	21st June	5.33
		3pm	21st December	107.5
Manchester	53.5	Noon	21st June	5.79
		3pm	21st December	150.7
Edinburgh	56.0	Noon	21st June	6.39
		3pm	21st December	302.16
Inverness	57.5	Noon	21st June	6.76
		3pm	21st December	759.54

Table 1: Shadow length of a tree with height 10m at 4 locations in the British Isles. (Lengths are shown for the longest day in summer and the shortest day in winter, giving the shortest and longest possible shadows for each location.)

The effect of topography

The above data for the length of shadow for a tree of height 10m assume that the tree is standing on flat ground with no variation in contours through to the horizon. Obviously, this is not often the case in the British Isles. As topography can be so varied, no general rule can be given for calculating shadow length but a few points should be kept in mind.

The site of the trees may be in the bottom of a valley or on the top of a hill, and it is important to take this into account for two reasons:

- If the site is on the floor of a valley running north/south, the surrounding hills may shade it from *direct* sunlight (which causes the shadows) until some time after sunrise. Likewise, direct sunlight may be lost some time before sunset. Therefore, there will be no long shadows, typical of early morning or late evening. But a valley running east/west *will* have long shadows.
- If trees are on top of a hill, they may cast shadows into a neighbouring valley (figure 4).

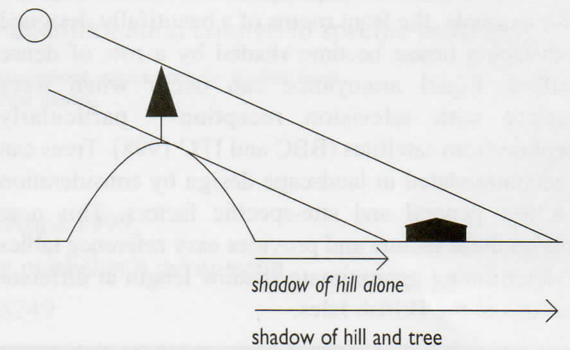

Figure 4.
Trees on top of a hill may increase shading in a neighbouring valley

Shadow lengths at different times and places

In table 2, shadow lengths for a tree of unit height (1m) have been calculated at London, Manchester, Edinburgh and Inverness at noon (2a) and 9am/3pm (2b). To determine the maximum shadow length for a tree of some fixed height in metres, these values have to be multiplied by the actual or expected tree height. The yearday is the number of the day in the year (January 1st = 001 and December 31st = 365 in a non-leap year).

Table 2: Shadow lengths (metres) for a tree of 1m height at each of London, Manchester, Edinburgh and Inverness. (All figures rounded to one decimal place).

2(a): shadow lengths at noon

		Location			
Date	**Yearday**	**London**	**Manchester**	**Edinburgh**	**Inverness**
1st January	1	3.6	4.2	5.1	6.0
1st February	32	2.5	2.8	3.3	3.6
1st March	60	1.7	1.8	2.0	2.1
1st April	91	1.1	1.1	1.3	1.3
1st May	121	0.7	0.8	0.9	0.9
1st June	152	0.6	0.6	0.7	0.7
21st June	172	0.5	0.6	0.6	0.7
1st July	182	0.5	0.6	0.6	0.7
1st August	213	0.7	0.7	0.8	0.8
1st September	244	0.9	1.0	1.1	1.2
1st October	274	1.4	1.5	1.7	1.8
1st November	305	2.1	2.4	2.7	2.9
1st December	335	3.0	3.4	4.1	4.6
21st December	355	3.7	4.3	5.4	6.3
31st December	365	3.6	4.2	5.2	6.0

2(b): shadow lengths at 9am or 3pm.

		Location			
Date	**Yearday**	**London**	**Manchester**	**Edinburgh**	**Inverness**
1st January	1	10.0	13.6	24.9	49.2
1st February	32	5.1	5.9	7.4	8.6
1st March	60	2.8	3.1	3.4	3.6
1st April	91	1.7	1.8	1.9	2.0
1st May	121	1.2	1.3	1.3	1.4
1st June	152	1.0	1.0	1.1	1.1
21st June	172	1.0	1.0	1.0	1.1
1st July	182	1.0	1.0	1.1	1.1
1st August	213	1.1	1.2	1.2	1.3
1st September	244	1.5	1.6	1.7	1.7
1st October	274	2.3	2.4	2.6	2.8
1st November	305	4.0	4.4	5.2	5.7
1st December	335	6.9	8.5	11.7	15.2
21st December	355	10.7	15.1	30.2	76.0
31st December	365	10.2	13.9	25.9	53.4

Tree shape and foliage density

A shadow is simply a projection of the tree shape onto the ground. Thus, a tall thin Douglas fir will cast a narrower shadow than an Oak of the same height with a broad crown. For information about mature tree height and branch spread see Hodge and White (1990). The Oak's crown will shade a building, for example, for much longer than the narrow crowned tree. Table 3 shows for how many hours a point at specific distances from the tree will be shaded. The *length* of the shadow has not been taken into account and must be calculated separately.

The density of the canopy should also be considered - Silver Birch trees have small leaves and let a lot of direct sunlight filter through their canopies, whereas evergreen oaks have dense canopies of leaves which shade the surrounding ground so that some plant species cannot grow beneath them. Deciduous trees lose their leaves in winter, and then the tree trunk and branches are the only parts which cast substantial shadows. Evergreen trees retain the same canopy density all year round.

Table 3: Figures in light type indicate the number of hours that a tree of specified crown width may shade a point at a given distance from the tree.

Duration of shadow in hours

Tree characteristics		**Distance from tree (metres)**			
Type	**Crown width in metres**	**5**	**10**	**15**	**20**
Rowan	**5**	3.5	1.9	1.3	1.0
Apple	**9**	5.6	3.2	2.2	1.7
Cypress	**12**	6.7	4.1	2.9	2.2
Birch	**14**	7.3	4.7	3.3	2.6
Maple	**18**	8.1	5.6	4.1	3.2
Oak	**20**	8.5	6.0	4.5	3.5

Ghosts and shadows on T.V.

Where there is concern about interference of television signals the same principles apply. By substituting the zenith angle of the satellite then shadow lengths can be calculated. Geo stationary satellites will have a constant shadow length for a given tree height. Where a deciduous tree is causing the shadow, the intensity of the shadow may vary between summer (tree in leaf) and winter (no leaves). (BBC and ITC (1998).

Shadows - what shadows?

Concern about the shading of windows and gardens by trees and interference with television reception are often cited as reasons for wanting trees severely pruned or felled. Also, new plantings are opposed because of possible shading the trees may cause.

Even a site visit can be of little help because the magnitude of any problem throughout the year cannot be evaluated fully. Decisions, therefore, have tended to be made on subjective or vague comments from clients and local residents.

Armed with information about latitude of a site, times of year and day as well as the height of a tree and its crown width - actual or projected - it is possible to calculate the magnitude of shading and assess its significance. Future conflicts can be handled more objectively and new plantings located so that conflict should not occur.

References and further reading:

BBC and ITV (1998) The Effects of Trees on Television Reception. *Arboriculture Research and Information Note 146/98/TV.* Arboricultural Advisory and Information Service, Farnham.

Dobson M.C., and Patch D (1998). Trees in Dispute. *Arboricultural Practice Note 3.* Arboricultural Advisory and Information Service, Farnham.

Hodge S.J., and White J.E.J. (1990). The Ultimate Size and Spread of Trees Commonly Grown in Towns. *Arboriculture Research Note 84/90/ARB.* Arboricultural Advisory and Information Service, Farnham.

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APPENDICES to Shaded By Trees?

The mathematical descriptions and procedures used to calculate the tables in the main text are presented below. With the aid of sine, cosine and tangent tables the formulae can be applied to specific problems and locations.

Appendix A: *The solar zenith angle*

The solar zenith angle, θ, is the angle between the sun and the sky zenith. It varies throughout the day and throughout the year. The variation can be described mathematically:

$$\cos\theta = \sin\phi \sin\delta + \cos\phi \cos\delta \cos t \qquad \text{(A1)}$$

Where ϕ is the latitude
δ is the sun's *declination* angle, i.e. how far north of the celestial equator it lies (Tables of values for every day of the year can be found in the Nautical Almanac).
t is the time of day expressed as an angle. t = 0 at midday and progresses 360/24 = 15 degrees each hour,

e.g.
1pm = 1x360/24 = 15 degrees
2pm = 2x360/24 = 30
6am = 18x360/24 = 270
10am = 22x360/24 = 330 etc

Example: Manchester - latitude, $\phi = 53.5°$

For 21st June, sun's declination, $\delta = +23.4°$ (i.e. north of the equator)
So for midday (t = 0), solar zenith angle equals
$\cos\theta = [\sin(53.5) \times \sin(23.4) + \cos(53.5) \times \cos(23.4) \times \cos(0)]$
$\theta =$ **30.1°**

For 21st December, sun's declination, $\delta = -23.4°$ (i.e. south of the equator)
So for midday (t = 0), solar zenith angle equals
$\cos\theta = [\sin(53.5) \times \sin(-23.4) + \cos(53.5) \times \cos(-23.4) \times \cos(0)]$
$\theta =$ **76.9°**

Appendix B: *The length of a shadow*

Given the solar zenith angle and knowing the height of the tree, the length of the shadow can be calculated.

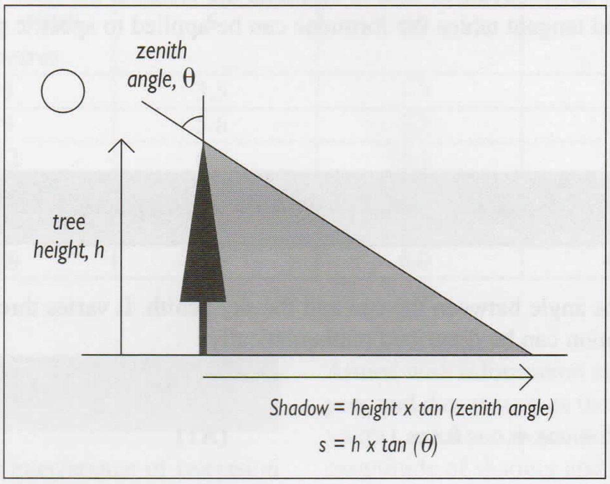

Figure B1: The shadow length *s* is given by multiplying the height of the tree *h* by the tangent of the solar zenith angle (tan θ).

Example: Take a 10m high oak tree in Manchester on 21st June at noon.
From appendix A, solar zenith angle, $\theta = 30.1°$
shadow length, s = height x tan (solar zenith angle)
= 10 x tan(30. 1)
shadow length, s = 5.8m

Appendix C: *The direction of the shadow*

For this, the *bearing* of the sun is needed (this is the angle between north and the sun).
The bearing B (north = 0°) is given by

$$B = 180° \times \frac{\text{time(decimal)*}}{12} \quad \text{(C 1)}$$

* *In decimal time 6 minutes = 0.1 hour*

For example, at 12 noon, bearing B will be 180°, i.e. the sun is in the south. Bear in mind that for the U.K., time has to be GMT - in the summer during British Summer Time, subtract one hour before doing the calculation. For the direction of the shadow, subtract 180° from the Sun bearing i.e. when the Sun is in the south (B = 180°) then the shadow will be to the north of the tree (180°- 180° = O°)

Example:
At 2pm, time = 14.0
Suns bearing B = 180° x 14.0/12 = 210°
Shadow will be at 210° - 180° = 30°

At 6.30 am, time = 6.5
Suns Bearing B = 180° x 6.5/12 = 97.5°
Shadow will be at 97.5° - 180° = 277.5°

Appendix D: *Calculating the time and direction of sunrise and sunset*

In order to find the bearing of the sun at sunrise and sunset, the solar zenith angle (Appendix A) has to be considered, i.e.:

(a) work out at what time the sun comes over the horizon (namely when θ equals 90°)

(b) use this time to work out the bearing (Appendix C)

(a) We can rearrange equation (Al) to give time:

$$\cos t = \frac{\cos\theta - \sin\phi \sin\delta}{\cos\phi \cos\delta} \qquad \text{(D1)}$$

Given that $\theta = 90°$ when the sun is rising or setting, the time t can be calculated. Remember that the cosine function is even - this means there are two solutions for the same cosine value

$$\cos(t) = \cos(360 - t) \qquad \text{(D2)}$$

In this case, t will be the sunset time and $(360-t)$ will be the sunrise time in degrees.

> *Example: Manchester, 21st June. Using values for ϕ and δ from appendix A, $\theta = 90°$ in equation (DI):*
>
> *$\cos t = \cos(360-t) = [(\cos(90) - \sin(53.5)\times\sin(23.4)) / (\cos(53.5)\times\cos(23.4))]$*
>
> *$\cos t = -0.59$*
>
> *This gives t (in degrees) = 126 degrees*
>
> *To convert to "real" time(!), t (hours after noon) = t (in degrees)/15*
> *=126/15*
> *=8.4 hours after noon*
> ***sunset = 20:24 GMT***
>
> *For sunrise, use (360 - t) = 360-126 = 234 degrees = 15.6 hours after noon*
> ***sunrise = 03:36 GMT***

(b) The bearing of the sun at sunrise and sunset is found by using equation (C 1) and the sunset and sunrise times as found above.

> *Example:*
>
> *Bearing of Sun at sunrise 03:36 B = 180° x(03:36 decimal time)/12*
> *=180° x (3.6/12)*
> ***Sunrise bearing = 54°***
>
> *Bearing of Sun at sunset 20:24 B = 180° x(20:24 decimal time)/12*
> *=180° x(20.4/12)*
> ***Sunset bearing = 306°***
>
> *To find the direction of the shadow, subtract 180°*

Appendix E: *The duration of the shading according to crown diameter*

This depends on crown width and distance from the tree at which the observer is standing.
The angle of view of the sky, A, which the tree blocks out is given by:

$$\mathrm{Tan}(A/2) = \frac{(crown\ diameter) / 2}{distance} \qquad \text{E1}$$

The duration of shadings, S, is proportional to this angle:

$$S = \frac{A}{360°} \times 24\text{hours} \qquad \text{E2}$$

Example:
Oak tree has crown diameter = 20m. Observer is standing 10m away.

$$\tan (A/2) = (10/10) = 1$$

$$(A/2) = 45°$$

$$\text{so } A = 2 \times 45° = 90°$$

$$\text{Duration of shading } S = (90° / 360°) \times 24 \text{ hours} = 6 \text{ hours}$$

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